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Essential work of fracture study of polymers: a novel criterion for the validation of tested ligament range

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Abstract The essential work of fracture method is widely used for the determination of fracture toughness of ductile metals and polymers under plane stress conditions. Nevertheless, this method has numerous prerequisites, which are easy to fulfill in metals, but are less certain in polymers. The aim of this study was, therefore, to present a simple, empiric "displacement-criterion" that helps the determination of valid ligament lengths for polymers—one of the biggest sources of error of this theory. This criterion not only facilitates the definition of a lower bound, but also helps the description of an upper ligament limit. Its other advantage is, that combined with a stress criterion, it helps the data validation not only until crack initiation, but also over the entire deformation process.

Introduction

The analysis of toughness of the ductile thermoplastic polymers remains a relevant issue, since the methods of linear elastic fracture mechanics (LEFM) fail due to the large plastic fields at the crack tip of such materials. One possible solution for this problem is the essential work of fracture (EWF) method [1].

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The EWF method was originally developed for the analysis of the fracture behavior of ductile metal sheets under plane stress conditions. The advantages of this concept compared with the other elastic–plastic fracture methods like J-integral or COD are the less complex test set-up, the ability to handle large scale yielding and the potential to trace the fracture behavior not only until crack initiation, but also through the crack propagation period [1, 2].

The original concept was that during stable crack growth, the total fracture work (W_f) is composed of a dissipative energy of the outer plastic zone (W_p) and of an essential work (W_e), which is required for the formation of new crack surfaces. The essential fracture work is assumed to be geometry independent, while the plastic fracture work depends on geometry, thus, the specific fracture work could be described by Eq. 1.

$$w_f = w_e + \beta w_p L, \quad (1)$$

where $w_f = W_f/LB$, $w_e = W_e/LB$, $w_p = W_p/L^2B$ and β is a geometry dependent correction factor, while L and B are the ligament length and the sheet thickness, respectively.

First of all, it should be mentioned that due to the extrapolation of data the sample size has a significant effect on the scattering of parameter values. Pegoretti et al. [3] analyzed the precision of EWF method as a function of sample size (N). They found that the standard error of the specific essential work of fracture decreases with the sample size (scaling as about $1/\sqrt{N}$). According to their article, a typical sample of 25 specimens results in a dispersion scale of 3–5%, which seems to be adequate for further considerations.

Furthermore, the test process should meet numerous requirements to be adequate for toughness determination.

First, the entire ligament has to yield before stable crack propagation. This can be ensured by reaching the yield stress in the entire ligament. Nevertheless, Hill [4] has shown that under plane stress conditions, the strain along a necked ligament for a plastic material must be zero, which results in a constraint that elevates the longitudinal stress by a factor of $2/\sqrt{3}$. Thus, as it was suggested by Cotterell et al. [2] the lower and upper bounds of maximum nominal stress ($\sigma_{\max} = F_{\max}/LB$) of a valid measurement should be the yield stress (σ_Y) and the $2/\sqrt{3}\sigma_Y$.

The other widely used stress criterion, proposed by Clutton [5], does not use the yield stress but determines the validity of tests empirically from the mean of the observed maximum stresses (σ_m)—see Eq. 2.

$$0.9\sigma_m < \sigma_{\max} < 1.1\sigma_m \quad (2)$$

The problem with these criteria is that they limit the fracture process only until crack initiation, while the propagation related stability problems are disregarded.

Second, as it was mentioned by Cotterell [2], one of the major sources of error in EWF method is the definition of valid ligament length. The lower bound is the plane-stress/plane-strain transition. This region is in the ligament range of 3–5B for metals [6]. In polymers, however, the different macromolecular deformation processes result in higher values. In linear low density polyethylene (LLDPE), the transition ligament length is assumed to be at 6–7B [7], while Wu and Mai [8] found even larger values, namely 14B. This inconsistency suggests that in polymers the plane-strain/plane-stress transition is material dependent and occurs at higher values.

The upper bound of ligament length is an asymptotic transition to the constant plane stress fracture toughness measured by LEFM [2]. In this case, the yielding of full ligament usually does not occur before crack propagation and the linear regression seems to be inappropriate [9, 10]. Thus, a rule of thumb is that the ligament length should not reach the third of specimen width (*W*/3) or the diameter of plastic zone ($2r_p$) [1].

The third—and unless a similarity transformation the most inexact—requirement of EWF method is the self-similarity of load displacement curves. This self-similarity allows the linear regression and extrapolation of measured results, but its adequateness strongly depends on the decisions of testing personnel, since the majority of research groups disregard the similarity transformation of load–displacement plots.

Martinez et al. [11] suggested a parameter—the ductility level (DL)—that measures the ratio of displacement at rupture (d_r) and original ligament length (DL $= d_r/L$). According to their theory, the studied materials were sorted into five groups, and the load–displacement responses of individual groups were self-similar; but strictly taken only two of the groups fulfilled the requirements of EWF tests. In addition, Gamez-Perez [12] found during the testing of polypropylene samples that DL increases with decreasing ligament lengths, which could be a result of the previously mentioned plane-stress/plane-strain transition. For the conventional valid ligament range, the DL values were nearly constant.

Based on these findings; in this article the feasibility of a simple, empiric criterion is addressed. The foundation of this hypothesis is the finding of Gamez-Perez [12, 13], namely the consistency of DL in valid ligament length range and its increase at small ligaments.

Experimental

In this study, compression molded poly(ε-caprolactone) (nominal number average molecular weight, $M_n = 80$ kDa) sheets with a thickness of 0.5 mm were analyzed. The double-edge notched tensile (DENT)—with a width of 40 mm and a clamped length of 40 mm—specimens were tested at a constant crosshead speed of 10 mm/min. Notches were prepared by aligned razor blades and the resulted ligament lengths varied between 1 and 28 mm. The yield strength used in Hill's criterion was obtained earlier, its value is $\sigma_Y = 17.02 \pm 0.23$ MPa [14]. The minimal sample size was set as $N = 25$, according to the results of Pegoretti et al. [3].

Results and discussion

Figure 1 shows a series of obtained load–displacement curves. As it is observable, the self-similarity holds in a wide ligament range. However, the DL values that are mainly

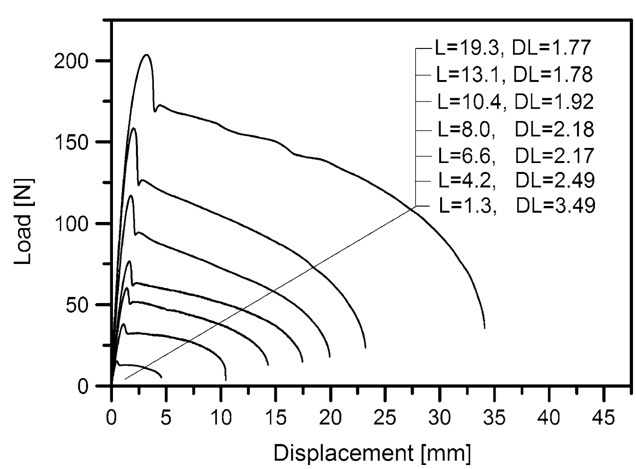

Fig. 1 Typical load–displacement curves with respective ligament lengths and DL values

related to the deformation processes during the stable crack propagation period—decreased fairly at higher ligament lengths, while increased at small ligament lengths—indicating the previously mentioned plane-strain transition.

Nevertheless, the majority of small, invalid ligament lengths could be filtered through the use of stress criterions of Cotterell [2] and Clutton [5] (Fig. 2). As it was mentioned earlier, the lower bound of Cotterell's criterion is the yield stress, 17 MPa, while the upper one is $\sim 1.15\sigma_Y$, which is 19.6 MPa. In Clutton's criterion, the mean of the observed maximum stresses is $\sigma_m = 18.5$ MPa, which yields in a range of 16.6–20.3 MPa after calculating $0.9\sigma_m$ and $1.1\sigma_m$. As a result, it turns out that for polymers Cotterell's criterion is stricter and narrower compared with that of Clutton's one. In addition, it is independent of the scatter of measured values, but requires further tensile tests under different stress conditions.

In Fig. 3, the obtained ductility levels (DL) are shown. The tendency at small ligament lengths and at the common ligament ranges is similar to the findings of Gamez-Perez [12]. However, at ligament lengths higher than $W/3$ the DL values start to decrease. This decreasing tendency is in connection with the asymptotic response assumed by Cotterell [2] and could lead to non-linearities during the regression as it was found by other authors [9, 10]. The problem, however, is that the original method fails when these non-linearities arise.

This observation suggests a simple empiric criterion based on the mean of the DL values, which helps the elimination of invalid data (Eq. 3).

$$0.9\overline{\mathrm{DL}} < \mathrm{DL} < 1.1\overline{\mathrm{DL}}, \quad (3)$$

where $\overline{\mathrm{DL}}\,[-]$ denotes the mean of ductility levels. Although this criterion is based on an averaging method—

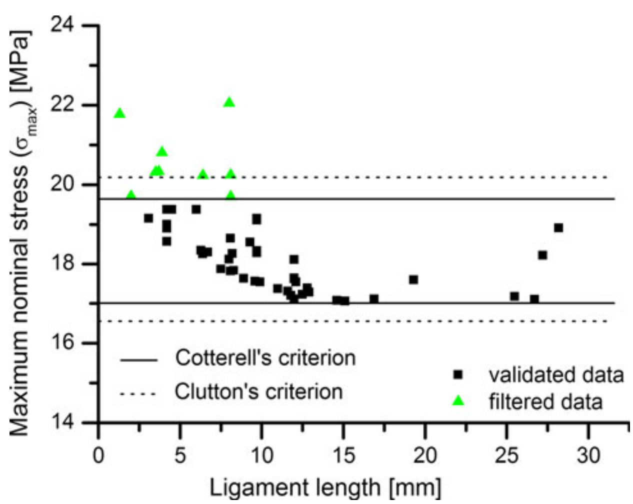

Fig. 2 Stress criterion of Cotterell [2] and Clutton [5] with the related bounds. "Validated data" denote data that have fulfilled the given criterion, while "filtered data" are those that have not

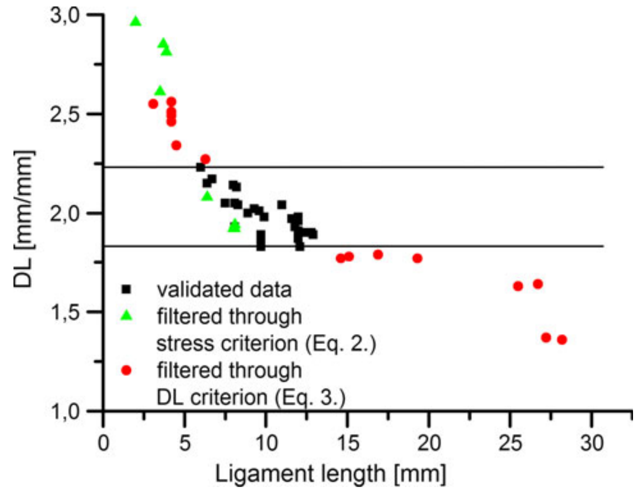

Fig. 3 Ductility level (DL) values as a function of ligament length/specimen width ratio

and therefore has similar material-dependence and drawbacks, like the empiric stress criterion of Clutton—it has a possible advantage; namely combined with a stress criterion, this "displacement criterion" allows a data-filtering through the whole fracture process, not just until initiation. In addition, not only the problem related to plane-stress/plane-strain transition is solved, but also the difficulties that arise from the non-linearities, incomplete yielding or contained plastic zone formation at large ligament lengths (greater than $W/3$).

Figure 4 shows the specific fracture works after the application of different data reducing methods. The first region is connected to the plane-stress/plane-strain transition, while the third region is linked to the asymptotic transition to plane-stress fracture toughness. It is also observable that the stress criterion itself works only well at

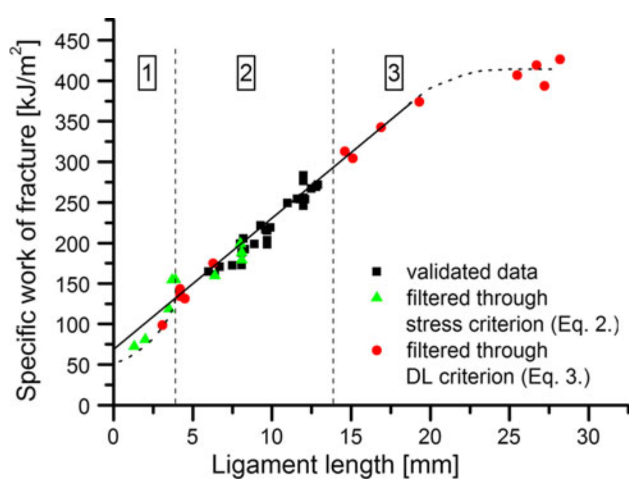

Fig. 4 Specific work of fracture values after the use of different criteria

Table 1 Essential work of fracture parameters and the goodness-of-fit of linear regressions

Data reducing method	Valid data range	w_e [kJ/m^2]	βw_p [MJ/m^3]	N [−]	R^2 [−]
Stress criterion [5]	$0.9\sigma_m < \sigma_{max} < 1.1\sigma_m$	91.3 ± 6.0	12.9 ± 0.5	43	0.9734
Criterion for metals [1, 2]	5B < L < W/3	68.4 ± 5.8	15.5 ± 0.6	40	0.9667
Criterion for LLDPE [7]	14B < L < W/3	43.9 ± 9.3	17.9 ± 1.0	32	0.9569
Equations 2 and 3	12B < L < W/3	49.2 ± 6.1	17.5 ± 0.8	29	0.9691
±2 Standard deviations	12B < L < W/3	60.8 ± 6.2	16.3 ± 0.6	22	0.9857

small ligament lengths. The errors resulting from greater ligament lengths are not handled by this method. For this purpose, the present displacement criterion seems to be more useful.

The incorrect determination of valid ligament lengths—lower and upper bounds—makes the original theory inappropriate and the non-linearities result in a significant deviation as it is shown in Table 1. The presented valid data range for polymers is narrower than that for metals and is in good correlation with the boundaries found by the other authors for similar polymers [7].

Related to the study of ethylene-propylene copolymer films, Williams and Rink [15] proposed a further criterion based on a statistical analysis. They stated that a correlation coefficient (R^2) of 0.98 or a standard deviation (S) value of 3 kJ/m^2 is required for correct data determination.

In this study, the observed R^2 values are close to 0.97 and seem to be adequate. However, the standard deviation values are quite high. Even after using Eqs. 2 and 3, the S value is close to 8 kJ/m^2. Nevertheless, the goodness of fit could be further improved by rejecting data of high residuals (outside of $\pm 2S$). Next, the regression line was recalculated until R^2 became higher than 0.98 and no data laid outside the range of $\pm 2S$—see Table 1 "±2 Standard deviations" line. After this final treatment, the number of data points slightly decreased ($N = 22$), but the calculated S/w_e ratio complied with the general 0.1 value [15].

The standard deviation, however, remained 6.3 kJ/m^2, which is nearly the double of $S = 3$ kJ/m^2 found for ethylene-propylene copolymers. This observation supports the statement of Williams and Rink [15], namely the S value seems to be material specific while the R^2 is not.

Conclusions

In summary, the aim of this study was to present a simple, empiric criterion for the validation of studied ligament lengths of EWF tests. The present displacement criterion seems to work well for PCL and the obtained ligament range is in good correlation with the limits derived by the other authors. Nevertheless, it should be noted that the restriction of valid ligament range might raise the possibility of errors emerging from the deviations in linear regression or from the incorrect determination of ligament length and thickness. Although the correlation coefficient can be improved by rejecting data outside of the standard deviation range, further studies are required to support the propriety of this hypothesis and to determine an appropriate minimum sample size.

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